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COSMOLOGY AND HIERARCHY IN STABILIZED RANDALL-SUNDRUM MODELS

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We consider the cosmology and hierarchy of scales in models with branes immersed in a five-dimensional curved spacetime subject to radion stabilization. The universe naturally find itself in the radiation-dominated epoch when the inter-brane spacing is static and stable, independent of the form of the stabilizing potential. We recover the standard Friedmann equations without assuming a specific form for the bulk energy-momentum tensor. We address the hierarchy problem in the context of a quartic and exponential stabilizing potential, and find that in either case the presence of a negative tension brane is required and that the string scale can be as low as the electroweak scale. In the situation of self-tuning branes (corresponding to an exponential potential) where the bulk cosmological constant is set to zero, the brane tensions have hierarchical values.

We summarize the results of Ref.[1] We assume the presence of three 3-branes in the space $(-\infty,\infty)$, at $y_0=0$, y_1 and y_2 with the observable brane located at y_0 . We will refer to the branes at y_1 and y_2 as hidden branes. The four-dimensional metric on the brane labelled by its position y_i is $g^{(i)}_{\mu\nu}(x^\mu)\equiv g_{\mu\nu}(x^\mu,y=y_i)$, where g_{AB} is the five-dimensional metric, $A,B=0,1,2,3,5$ and $\mu,\nu=0,1,2,3$. The five-dimensional Einstein equations including a radion field ϕ are, $G_{AB}=\kappa^2\,T_{AB}=\frac{1}{2}\,(\partial_A\phi\,\partial_B\phi-\frac{1}{2}\,g_{AB}\,(\partial\phi)^2)-\kappa^2\,g_{AB}\Lambda(\phi)-\kappa^2\,\sum_{i=0}^{2}V_i(\phi)\sqrt{\frac{g^{(i)}}{g}}\;g^{(i)}_{\mu\nu}\;\delta^\mu_A\delta^\nu_B\;\delta(y-y_i)$, where $\kappa^2=8\,\pi\,G_N^{(5)}=M_X^{-3}$ is the five-dimensional coupling constant of gravity and M_X is the Planck scale in five dimensions. $V_i(\phi(y_i))$ is the tension of the brane at y_i and $\Lambda(\phi)$ is the potential of the radion in the bulk and is interpreted as the cosmological constant although it has a ϕ-dependence. We allow it to be discontinuous at the branes, but continuous in each section. We write $\Lambda(\phi)$ as $\Lambda_0(\phi)$ if $y<0$, $\Lambda_1(\phi)$ if $0<y<y_1$ as $\Lambda_2(\phi)$ if $y_1<y<y_2$ and $\Lambda_3(\phi)$ if $y>y_2$.

The most general five-dimensional metric that respects four-dimensional Poincaré symmetry is $ds^2=e^{2\,A(y)}\;\eta_{\mu\nu}\;dx^\mu dx^\nu+(dy)^2$. Then Einstein's equations can be written as[2] $2\,\kappa^2\,\Lambda(\phi)=\frac{1}{2}\,(\frac{\partial W(\phi)}{\partial\phi})^2-\frac{1}{3}\,W(\phi)^2$, $\phi'=\frac{\partial W(\phi)}{\partial\phi}$, $A'=-\frac{1}{6}\,W(\phi)$, subject to the constraints $W(\phi)\Big|_{y_i-\epsilon}^{y_i+\epsilon}=2\,\kappa^2 V_i(\phi_i)$, $\frac{\partial W(\phi)}{\partial\phi}\Big|_{y_i-\epsilon}^{y_i+\epsilon}=2\,\kappa^2\,\frac{\partial V_i(\phi_i)}{\partial\phi}$, where $\phi_i\equiv\phi(y_i)$ and $W(\phi)$ is any sectionally continuous function (which we call the superpotential), with sectional functions $W_i(\phi)$ defined analogous to $\Lambda_i(\phi)$.

With this formalism in place, we turn to cosmology. Consider a metric of the form $ds^2=-n^2(\tau,y)d\tau^2+a^2(\tau,y)d\mathbf{x}^2+dy^2$, which encodes our assumption of a static stabilizing potential. We can study the contribution of matter energy densities on the observable brane as a perturbation to the brane tension by making the

ansatz $\rho_0 = \rho + V_0$, $p_0 = p - V_0$, where functions with the subscript 0 are evaluated on the observable brane, and ρ and p are the perturbations. By requiring e^{2A} to be symmetric on either side of the observable brane, *i.e.*, $W(\phi(+\epsilon)) = -W(\phi(-\epsilon))$, it is possible to show that $\left(\frac{\dot{a_0}}{a_0}\right)^2 + \frac{\ddot{a_0}}{a_0} = \frac{\kappa^4}{36} V_0 (\rho - 3p) - \frac{\kappa^4}{36} \rho (\rho + 3p)$, where we have used $\kappa^2 \langle \check{T}_{55} \rangle = \frac{1}{6} \langle W(\phi_0)^2 \rangle = \frac{\kappa^4}{6} V_0^2$ and $2 \langle \check{T}_{55} \rangle = \check{T}_{55}(+\epsilon) - \check{T}_{55}(-\epsilon)$. The leading term on the right-hand side reproduces the standard cosmology if we make the identification, $\kappa^4 V_0 = 6/M_{Pl}^2$. Note that a specific form of the bulk energy-momentum tensor was not chosen in the derivation. In introducing the perturbation, it is no longer obvious that the equation of motion of ϕ remains consistent. On requiring that ϕ and $\Lambda(\phi)$ be unchanged before and after the introduction of the matter energy density, consistency requires, $\left(3 \frac{a'}{a} + \frac{n'}{n}\right)\Big|_{0\,,\,Static} = \left(3 \frac{a'}{a} + \frac{n'}{n}\right)\Big|_{0\,,\,Perturbed}$, which is the condition for a radiation-dominated (RD) universe, $\rho = 3p$. This result is consistent with the fact that the radion couples to the trace of the energy-momentum tensor. It may be possible to identify the process of radion stabilization with inflation and reheating and the time at which the inter-brane spacing becomes stable marks the end of reheating. The RD universe then ensues.

Let us address the hierarchy problem in the context of two stabilizing potentials that have received considerable interest.[3,4] Consider a superpotential of the exponential form $W_i(\phi) = \omega_i \, e^{-\beta\phi}$, for which $12\,\kappa^2 \Lambda_i(\phi) = (3\beta^2 - 2)\,\omega_i^2 \, e^{-2\beta\phi}$. For $\beta^2 = 2/3$, we have the important result that $\Lambda = 0$.[3] With $\beta^2 = 2/3$ the branes are flat and will remain so, independent of the matter on them (hence the expression "self-tuning flat branes"). It can be shown that $f(y) \equiv e^{\beta\phi(y)} = -\frac{2}{3}\,\omega_i \, y + c_i$ for $y_{i-1} \leq y \leq y_i$, and $e^{2A(y)} = \sqrt{e^{\beta\phi}}$. Note that when f vanishes, ϕ diverges and e^{2A} vanishes. We truncate the space at the horizons defined by $e^{2A} = 0$. We can calculate the four-dimensional Planck scale in terms of the five-dimensional Planck scale,[5] $M_{Pl}^2 = M_X^3 \int e^{2A(y)}\, dy$. The result is $M_{Pl}^2 = M_X^3 \left[(\frac{1}{\omega_1} - \frac{1}{\omega_0})\, e^{\phi_0/\beta} + (\frac{1}{\omega_2} - \frac{1}{\omega_1})\, e^{\phi_1/\beta} + (\frac{1}{\omega_3} - \frac{1}{\omega_2})\, e^{\phi_2/\beta} \right]$. We recall that the electroweak scale (M_{EW}) can be generated from the five-dimensional Planck scale M_X via $M_{EW} \simeq M_X \, e^{A(0)} = M_X \, e^{\phi_0/(6\beta)}$. In a two brane geometry, imposing the cosmological requirement that $W(\phi)^2$ be symmetric about the observable brane results in ϕ always taking its maximum value on the observable brane thus making it impossible to solve the hierarchy problem. We have shown[1] that Fig. 1 displays the unique minimal configuration from which the hierarchy of scales can be obtained without fine-tuning. There are two positive tension branes (one of which is the observable brane), and one negative tension brane. By inspecting $12\,\kappa^2\, V_i(\phi) = (\omega_{i+1} - \omega_i)\, e^{-\beta\phi}$ it can be seen that due to the exponential dependence of the brane tensions on ϕ , a large hierarchy is generated between the values of the tensions for even moderately different values of ϕ.

If we consider the type of superpotential that leads to the stabilization mechanism suggested in Ref.[4], we find that it is not possible to generate the appropriate scale hierarchy with only positive tension branes and that it is necessary for the radion to be unbounded for $y > y_1$. We therefore study a model with two branes

where the hidden brane has negative tension. Table 1 shows our particular choice of the polynomial superpotential and the solution to Einstein's equations. The location of the hidden brane is $y_1 = \frac{1}{2}\ln\frac{\phi_1}{\phi_0}$, the electroweak scale is $M_{EW} \simeq M_X\, e^{-\frac{\phi_0^2}{24}}$ and the Planck scale is given by $\left(2\,\frac{M_{Pl}}{M_X}\right)^2 = \left(\frac{\phi_0^2}{12}\right)^{-\frac{\xi}{12}}\Gamma\left(\frac{\xi}{12},\frac{\phi_1^2}{12},\infty\right) + \left(\frac{\phi_0^2}{12}\right)^{-\frac{\eta}{12}}\left[\Gamma\left(\frac{\eta}{12},\frac{\phi_0^2}{12},\frac{\phi_1^2}{12}\right)+\Gamma\left(\frac{\eta}{12},\frac{\phi_0^2}{12},\infty\right)\right]$, where $\Gamma(a,x,y) \equiv \int_x^y t^{a-1}\, e^{-t}\, dt$. Consistency conditions imposed by positivity of the tension of the observable brane and the profile of the radion are $\eta < \phi_0^2 < \phi_1^2$. When the correct hierarchy is generated, by far the dominant contribution to M_{Pl} comes from the integral over the space $y > y_1$. The condition under which this integral dominates is $\phi_0^2 < \phi_1^2 < \xi$. Then $\eta < \xi$, and the brane at y_1 has negative tension.

For both the potentials considered, it is not possible to place the negative tension brane at the fixed point of an orbifold because the space beyond y_1 is crucial for generating the scale hierarchy. As a result of this, the radion may have a problem with positivity of energy. If one accepts this unpleasant circumstance the models are theoretically feasible.

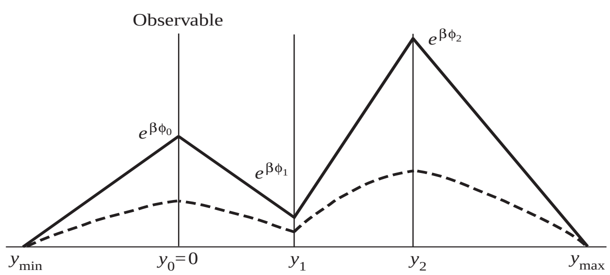

Fig. 1. The profile of $e^{\beta\phi}$ (solid) and e^{2A} (dashed) in the case of self-tuning branes. In regions where ω_i is positive (negative), $e^{\beta\phi}$ falls (rises) linearly. The brane at y_1 has negative tension.

$W(\phi)/M_X$	$\phi(y)$	$A(y)$	Region
$(\eta-\phi^2)$	$\phi_0\, e^{2\,\lvert y\rvert}$	$\frac{1}{6}\,\eta\,\lvert y\rvert - \frac{\phi_0^2}{24}\, e^{4\,\lvert y\rvert}$	$y<0$
$-(\eta-\phi^2)$	$\phi_0\, e^{2\,y}$	$\frac{1}{6}\,\eta\, y - \frac{\phi_0^2}{24}\, e^{4\,y}$	$0<y<y_1$
$-(\xi-\phi^2)$	$\phi_0\, e^{2\,y}$	$\frac{1}{6}\,\eta\, y_1 + \frac{1}{6}\,\xi\,(y-y_1) - \frac{\phi_0^2}{24}\, e^{4\,y}$	$y_1<y$

Table 1: The solution to Einstein's equations in a model with a polynomial superpotential.

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